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Particles on $AdS_{4/7}$ and primary operators on $M_{2/5}$ brane worldvolumes

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Abstract: I identify a correspondence between the various spherical harmonic modes of massless 11 dimensional fields propagating on the $AdS_{4/7}$ in an $AdS_{4/7} \times S^{7/4}$ compactification of M theory, and the corresponding operators, primary under the conformal group, on the world volume of the M_2, M_5 branes. This is achieved by matching representations of the superconformal algebra on the two sides of the correspondence.

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Contents

1. Introduction

Recently Maldacena [1] observed that the duality between the 'solution to low energy equation of motion' and gauge theory (D brane) descriptions of solitons, implies a connection between the world volume theory of some p branes, and M (or string) theory on $AdS_{p+2} \times S^{D-p-2}$. In particular he conjectured that the world volume theory of the $M_{2/5}$ brane is dual to M theory on $AdS_{4/7} \times S^{7/4}$.

Accepting the validity of his arguments, the next obvious task is to make the statement of the proposed duality more precise. This involves identifying a map between operators on the (superconformal) world volume field theory and objects in the bulk theory, and stating the precise nature of this map (correlation functions on the world volume theory map onto what?) These tasks have been partially accomplished for the special case of the D_3 brane in [2, 3, 4, 5] among others

In this paper I identify a correspondence between the various spherical harmonic modes of massless 11 dimensional fields propagating on the $AdS_{4/7}$ in an $AdS_{4/7} \times S^{7/4}$ compactification of M theory, and the corresponding operators, primary under the conformal group, on the world volume of the M_2, M_5 branes. This is achieved by matching representations of the superconformal algebra on the two sides of the correspondence.

The contents of the paper are as follows. In section 2 I state some properties of representations of superconformal algebras, and introduce special representations that will be of interest to us. In section 3 I review papers on the particle content of supergravity compactifications on the background of interest, and note the results of [6, 7], grouping these particles into the superconformal representations reviewed in section 2. In section 3 I identify the superconformal primary operators on the world volume of the M_2/M_5 branes that lead to same representations of the superconformal algebra as those obtained on analyzing the representation content of particles particles propagating on AdS space in the corresponding supergravity theory. Descendents of these superconformal primary operators that are conformal primary operators correspond to particles on the AdS space. I give a reasonably easy to implement algorithm to determine an explicit expression for the world volume operator corresponding to any specific particle on the AdS spaces.

2. Special representations of the relevant superalgebras

Unitary representations of the $D = 3, \mathcal{N} = 8$ and $D = 6, n = 2$ superconformal algebras have been studied. Representations are infinite dimensional, and are completely characterized by a lowest weight multiplet of states. Lowest weights states appear in multiplets of $SO(3) \times SO(8)$ and $SO(6) \times SO(5)$ in the two cases above respectively, and are labeled by the representations of these groups into which they fall, along with a number ϵ_0, the scaling dimension of this representation. The condition of unitarity of representations imposes an inequality, $\epsilon_0 \geq f(group \;\; rep)$, on the scaling dimension of allowed lowest weight states. The precise form of these inequalities has been derived in [8]. Representations at values of ϵ_0 saturating the inequality above, and especially those representations that are at isolated allowed values of ϵ_0, are special short representations, containing fewer states than a generic representation. They are the analogues of BPS representations of Poincare supersymmetry. Examples of such representations are

a) Rep $Chiral_3(k)$: $(D = 3, \mathcal{N} = 8)$. $\epsilon_0 = \frac{1}{2}k$; $SO(3) = scalar$; $SO(8) = (k, 0, 0, 0)$[1] in a convention (followed by [6]) in which supersymmetry generators transform as $SO(8)$ chiral spinors.[2]

b) Rep $Chiral_6(k)$: $(D = 6, \mathcal{N} = 2)$. $\epsilon_0 = 2k$, $SO(6) = scalar$ $SO(5) = (k, 0)$.

$Chiral_3(k)$ and $Chiral_6(k)$ are ultrashort representations of the relevant superconformal algebras, occurring at isolated values of allowed ϵ_0 as deduced in [8]. These particular representations will turn out to be of importance to us.

[1] I always specify $SO(k)$ weights in the GZ labeling system, in which the vector is (1, 0, 0..), the spinor $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}...)$, etc

[2] In [8] scalar ultra short representations were derived to have scaling dimension $\epsilon_0 = h_1$ where (h_1, h_2, h_3, h_4) denoted the $SO(8)$ representation. In that paper, however, supersymmetry generators were taken to transform in the vector of $SO(8)$. To make contact with the results of that paper one makes a triality transform , under which $(k, 0, 0, 0) \longrightarrow (\frac{k}{2}, \frac{k}{2}, \frac{k}{2}, \frac{k}{2})$.

Gunaydin and collaborators have developed an oscillator technique to construct representations of Lie Super Algebras. The representations above turn out to be particularly easy to construct using this method. The construction has been performed in [6, 7], for $Chiral_3(k)$ and $Chiral_6(k)$ respectively.

Any representation of the superconformal algebra may be decomposed into the sum of finitely many representations of the conformal algebra.[3] Gunaydin and collaborators have performed this decomposition for the representations listed above. The results of this decomposition is listed in Table 1 in each of [6, 7].

3. Superconformal representations and the compactification of d=11 SUGRA on $AdS_{4/7} \times S^{7/4}$

Consider M theory compactified on $AdS_{4/7} \times S^{7/4}$. Purely classically, the massless states of M theory in 11 dimensions lead to infinite towers of 'particles' in the 4/7 dimensional AdS spaces. Killing vectors of $AdS_{4/7}$ generate killing symmetries of $SO(3,2)/SO(6,2)$. Equations of motion for all 4/7 dimensional particles commute with killing vectors; thus on shell modes of particles transform in representations (typically irreducible) of the $SO(3,2)/SO(6,2)$ killing symmetry group. Therefore on shell wave function modes of elementary particles on $AdS_{4/7}$ appear in irreducible representations of $SO(3,2)/SO(6,2)$.

The net symmetry supergroup of the whole 11 dimensional space is $SO(3,2/8)/SO(6,2/2)$,[4] and so all modes on the compactified 11 dimensional spacetime must appear in multiplets the $(D=3, \mathcal{N}=8)/(D=6, \mathcal{N}=2)$ superconformal algebras, SO(3,2/8)/SO(6,2/2). Since any representation of a superconformal algebra may be decomposed into a sum of representations of the corresponding conformal algebra, and since each particle on this space constitutes a representation of the $d=3,6$ conformal algebra, particles must appear in families such the on shell wave function of each family of particles constitutes an irreducible representation of the d=3/6 superconformal algebras.

d=11 supergravity on $AdS_{4/7} \times S^{7/4}$ has been studied, and the resulting $d=4/7$ particle content extracted in [9, 10]. The particles thus obtained may be grouped into supermultiplets of the relevant superalgebras (this observation was made in [6, 7]). The representations of the superconformal algebras that these resulting particles lie in are all of, and only $Chiral_3(k)/Chiral_6(k)$, with $k \geq 2$.

References [6, 7] explicitly decompose Reps $Chiral_3(k)/Chiral_6(k)$ into representations of the conformal group; to each such representation is a devoted a row in Table 1

[3]In analogy with the fact that representations of supersymmetry consist of finitely many irreducible representations of the Poincare algebra, ie finitely many particles. BPS representations are small because they contain few particles. Similarly ultrashort representations of the superconformal algebra are short because they contain few multiplets of the conformal algebra

[4]The $SO(8)/SO(5)$ R symmetry is the killing symmetry of the sphere

of these references. Since particles on $AdS_{4/7}$ are irreducible representations of the relevant conformal algebra, there must be a one one correspondence between d=4/7 dimensional particles obtained from supergravity and rows in Table 1 of [6, 7] for $k \geq 2$.

The $d = 4/7$ particles obtained by a compactification of $d = 11$ SUGRA on $AdS_{4/7} \times S_{7/4}$ have been listed in [9, 10]. One may try to establish a correspondence between the lists of particles in these references, and the rows of Table 1 of [6, 7].

This correspondence has already been noted and made explicit in Table one of [7] in the $d = 7$ case. In the case $d = 4$ the explicit correspondence may be made by comparing Table 3 of [9] and Table 1 of [6]; specifically by comparing (column 1 , column 2) in table 3 of [9][5] with (column 3, column 5) of [6]. When comparing note that (n of [9]) = (n of [6]) minus 2.

4. The world volume theory of the $M_{2/5}$ brane

$M_{2/5}$ branes are governed by a world volume theories that are superconformally invariant. Some information on these theories may be found in [11].

The theory on the world volume of m coincident M_2 branes is the infrared limit of the world volume theory of m coincident D_2 branes. The (0,2) theory of the M 5-brane may also be connected to a gauge theory via flow in energy scale as follows. Consider an M_5 brane wrapped around the M theory circle of radius R in the decoupling limit (M_{11} taken to ∞) at energy scale E. At low energies (or at fixed energy as $R \to 0$) the branes are governed by the 4 dimensional world volume theory of m D_4 branes at weak effective coupling $= g_{ym}^2 E = RE$. At large E (or fixed E and large R) the world volume theory is effectively that of the uncompactified (0,2) theory.

Consider the world volume theory of a single D_2 brane. World volume fields consist of 7 free real scalars, 8 free real spinors and a single abelian gauge field. In 3 dimensions the abelian photon is dual to a compact scalar. In the infrared limit, the compactification radius of the scalar goes to infinity, so in the infrared a single D_2 brane is described by 8 real scalar and 8 real spinor degrees of freedom. The scalars are taken to transform in the vector of $SO(8)$. The fermions transform in the antichiral spinor of $SO(8)$. Supersymmetry generators transform in the chiral spinor of $SO(8)$.[6] The Supersymmetry transformation laws are

$$\delta\lambda^{\tilde{\alpha}} = \frac{i}{2}\gamma^{\mu}\Gamma^{a}_{\alpha\tilde{\alpha}}\epsilon^{\alpha}\partial_{\mu}\phi^{a} \tag{4.1}$$

$$\delta\phi^{a} = \bar{\epsilon}^{a}\Gamma^{a}_{\alpha\tilde{\alpha}}\lambda^{\tilde{\alpha}}\,. \tag{4.2}$$

[5] Note also formulae 3.10 and 3.11 in [9]. These formulae list the mass of all 5 dimensional particles obtained on compactification as a function of the scaling dimension of the associated representation of the conformal group (what is referred to in that reference as the 'Anti De Sitter Energy' of the representation). E_o in that paper is what I have called ϵ_0.

[6] To keep conventions those of [6]

The interacting theory of m coincident D_2 brane may similarly be regarded as a theory of 8 matrix valued scalar fields, one of which is 'compact' (and superpartners), even though in this case the dualization of the photon into a scalar cannot be explicitly performed.

In the interacting theory, the operators $Tr(X^{i_1}..X^{i_k})$, (where $i_1..i_k$ running from 1..7 are symmetrized and traceless) transform in the representation $(k,0,0..)$ of $SO(7)$, the internal symmetry group of $N=8$ SYM. The internal symmetry group of the theory is enhanced to $SO(8)$ in the infrared. Therefore the above multiplet of operators must pair up with some other operators (consisting of symmetrized traces of Xs and dualized photons) to form multiplets of $SO(8)$ in the infrared, transforming in $(k,0,0..0)$ of that group. I denote these operators by $Tr(X^{j_1}..X^{j_k})$, where $j_1..j_k$ run from 1..8. I emphasize that by $Tr(X^{j_1}..X^{j_k})$ I mean the operators to which these objects flow in the infrared - the actual infrared theory may have no convenient formulation in terms of $U(m)$ matrices of scalar fields.

I now conjecture that, at the infrared fixed point, these operators are superconformal primary, and that their scaling dimension are given by $\epsilon_0 = \frac{k}{2}$. This implies that these operators head a superconformal multiplet of operators transforming in representation $Chiral_3(k)$ of the $D=3, \mathcal{N}=8$ superconformal algebra. These operators and their descendents, therefore, correspond to the various modes of particles in the Kalutza Klein compactification of M theory on $AdS_4 \times S^7$.

M theory on $AdS_4 \times S^7$ also has multi particle supergravity states. These states presumedly correspond, on the world volume of the M_2 brane, to products of the operators considered above. As m is taken to infinity however, the effective Planck Mass of the M theory on $AdS_4 \times S^7$ goes to infinity, and presumedly all nonsupergravity states attain infinite mass. This implies that the operators considered above along with their products exhaust the space of operators on the world volume theory at $m=\infty$.

Notice that the D_2 brane theory certainly possesses gauge invariant operators other than those considered above, for instance $Tr(X^iX^i)$, whose scaling dimension at very high energies (at which the gauge theory is free) is 1. The AdS correspondence predicts that the scaling dimension of this operator (along with many others) runs off to infinity at low energies and $m=\infty$. This statement is plausible, because the effective (t'Hooft) coupling of the theory scales like m, and so the large m theory is very strongly coupled.

The story with the M_5 brane is similar. The microscopic fields of the free theory of a single M_5 brane have 5 scalars ϕ^a transforming as an $SO(5)$ vector, a single self dual two form field $B_{\mu\nu}$, and 4 chiral spinors λ, transforming under $SO(5)$ as a chiral spinor. An explicit action for this theory may be found in section 4 of [12] for instance. The supersymmetry transformation properties of the free theory are listed in [12]. The fields above transform under Rep $Chiral_6(1)$ of $SO(6,2/2)$.

The theory of m coincident D_4 branes possesses operators $Tr(X^{i_1}..X^{i_k})$ (with $i_1..i_k$ running from 1..5, and completely symmetrized and traceless) in the representation $(k,0,0..0)$ of $SO(5)$. At very low energies these operators have scaling dimension $\frac{3}{2}k$.

I conjecture that these operators on the D_4 brane world volume are connected , via the renormalization group flow described at the beginning of this section, to primary operators of scaling dimension $2k$ on the $(0,2)$ theory. These operators head the infinite dimensional representation $Chiral_6(k)$ for all $k > 1$, of the $d = 6, (0,2)$ superconformal algebra, and correspond to the supergravity particles propagating on $AdS^7 \times S^4$ described in section 3.

The absence of other operators (save products of these) in the spectrum of the infinite m theory follows as for the 3 dimensional case.

The spectrum of operators obtained in this theory agrees with the results of the DLCQ performed in [13], although the truncation of the spectrum of operators observed in [11] does not emerge in any obvious fashion from supergravity, and is probably related to a stringy exclusion principle [14].

In the next section I will denote primary operators by a trace over ϕ fields. This notation refers to the primary operators on $M-branes$, connected to the specified trace operators in the relevant gauge theory by renormalization group flow. Conservatively this description may be thought of as mere notation to keep track of quantum numbers.

5. Detailed operator particle map

Having identified the primary operators under the superconformal group corresponding to superconformal representations that occur in the $d = 4/7$ particle spectrum of the previous section, it is now easy to make a correspondence between specific particles on $AdS_{4/7}$ in the supergravity and specific descendents of one of these operators. Since our primary operators are lowest weight states in representations of the superconformal algebra that are identical to those constructed in [6, 15], one may obtain all conformally primary descendents of the superconformal primary operators by a procedure identical to the one used in [6, 7] in generating lowest weight states of the conformal algebra from those of the superconformal algebra. Examining the procedure adopted in those papers leads to the following algorithm

Consider the (superconformal) primary operators that emerge from the trace over k ϕ fields. Symmetrize the $SO(8)/SO(5)$ indices of all these fields. This product is a primary operator under the superconformal algebra with as many descendent operators that are primary operators of the conformal algebra as there are entries (for the relevant k) in table 1 of [6, 7]. Every entry (relevant to that k) in the table is associated with a specific conformal primary operator. To construct the field corresponding to a given entry in that table, act on the product of ϕs above with as many (n) Q operators as there are boxes in either of the Young tableau appearing in column 2/1 of that table. Symmetrize the $SU(2)/SU(4)$ vector indices on the Qs according to the first of the young tableau in column 2/1 of the table. Symmetrize the $SO(8)/SO(5)$ indices of the

Qs according to the second of the young tableau in column 2/1 of the table.[7] Multiply the Q and ϕ indices by $SO(8)/SO(5)$ Clebsch Gordan coefficients, and sum, so as to project onto the $SO(8)/SO(5)$ representation listed in column 4(or 5)/ 2(or 7) of that table. The operator thus obtained is a primary operator of the conformal group, with scaling dimension $\epsilon_0 = \frac{n+m}{2}$ / $2n + \frac{m}{2}$, $SO(3)/SO(6)$ transformations as given by the first Young Tableau in column 2/1 of the table, and appearing in $SO(8)/SO(5)$ multiplets listed in columns [4 and 5] /[2 and 7] of the table.

I elaborate on this process for some simple examples to clarify procedure. Consider the AdS_4 graviton. This corresponds to the only spin 2 particle in Table 1 of [6]. Making reference to the mass formula in [9], we note that the massless graviton corresponds to $k = 2$ in [6]. Using Table 1 in that reference, we thus see that the graviton is obtained by acting the product of 4 Q operators on the trace of two ϕ fields. According to the table and the algorithm above, the 4 $SU(2)$ spinor indices on these Qs must be completely symmetrized. That makes sense as it yields a spin 2 particle. According to the table, the 4 $SO(8)$ chiral spinor indices on the Qs must be completely antisymmetrized. This process leads to two $SO(8)$ representations, the symmetric tensor, and the anti selfdual 4 form. Since we are (according to the table) supposed to couple this representation to the symmetric tensor (from the ϕs) to form a scalar, we select out the symmetric tensor, and contract with the symmetric tensor indices on ϕs to get an $SO(8)$ scalar. Following through this procedure, and using the explicit expressions above for the action of the supersymmetry generators on fields yields an explicit expression for the world volume field corresponding to the $d = 4$ graviton.

As another example consider the AdS_7 massless vector particle. It corresponds to the entry in row 4 of Table 1 in [7]. It occurs at $p = 2$ in that table (according to the mass formula in the table). According to the table, it is formed by acting on the trace on a product of 2 ϕs by 2 Qs. Antisymmetrize the chiral $SO(6)$ spinor indices on these Qs obtaining an $SO(6)$ vector. Symmetrizes the $SO(5)$ spinor indices on these Qs obtaining an antisymmetric tensor. Finally Clebsch Gordan couple the antisymmetric tensor with the symmetric tensor (from the ϕs) to get an antisymmetric tensor (according to the table).

As a last example consider the AdS_7 graviton. It corresponds to row 6 in Table 1 of [7], $p = 2$. One obtains it by acting on the trace of 2 ϕs with 4 Qs. The Q $SO(6)$ indices are pair wise anti symmetrized to form 2 sets of $SO(6)$ vector indices, and these vector indices are mutually symmetrized. The Q $SO(5)$ spinor indices are also pairwise antisymmetrized to form $SO(5)$ vectors (discard the scalar component), but the pairs are chosen incommensurately with the first set of pairs. The resulting 2 $SO(5)$ vector indices are then symmetrized to form a symmetric tensor, which is then contracted with the symmetric tensor indices of the ϕs to yield an $SO(5)$ scalar

[7]The relative symmetrization of Lorentz and R symmetry indices should be opposite - ie chosen such that the product of Qs is completely antisymmetric

6. Conclusion

In this paper I have mapped chiral superconformal primary operators of world volume theories onto groups of particles appearing in the corresponding Maldacena dual SUGRA on $AdS_{4/7} \times S^7$. In the process, I have computed the spectrum of chiral operators on the worldvolume of the $M_{2/5}$ brane. The spectrum so obtained agrees with the analysis of [13] for the M_5 brane, and so may be regarded as a test of Maldacena's conjecture.

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